1. (30 points) For the system below
   a.) Find the value of \( K \) that will cause the steady-state value of \( C(s)/D(s) \) to be less than 2 percent of the steady-state value of \( C(s)/R(s) \), based on a unit step function input to both \( D(s) \) and \( R(s) \).

\[
\frac{C}{D} = \frac{1}{\frac{s+12}{s+10} + \frac{12K}{s+10(s+2)}} = \frac{s+10}{s^2+12s+20+10K}
\]

\[
\frac{C}{K} = \frac{(s+10)K}{s^2+12s+20+10K}
\]

\[
\left| \frac{C}{R} \right|_{ss} = \left| \frac{1}{K} \right| \Rightarrow K = 50
\]

b.) Find the value of \( K \) which will result in a steady-state error of less than 2 percent, based on a unit step function input to \( R(s) \), assuming \( D(s) = 0 \) and the output is desired to match the input.

\[
\frac{C}{R} = \frac{(s+10)K}{s^2+12s+20+10K}
\]

\[
C_{ss} = \frac{10K}{s+10k} = \frac{K}{2+K} = 0.98
\]

\[
K = 98
\]

\[
K = 1.96 + 0.98K
\]
2. (60 points) You are attempting to design a PID control system and you initially set \( K_i = 0 \), \( K_p = 2 \) and \( K_d = 1 \). In order to evaluate the performance of this system, please find all of the attributes below.

a.) Give the values of \( K \) for which the system is stable.

\[
\frac{C(s)}{R(s)} = \frac{K(s+2)}{s^2 K(k-2)s + 2(k+1)}
\]

From this \( K > 2 \) (5 pts)

b.) Tell what the minimum settling time will be and whether the system will be oscillatory there.

\[
S = \frac{-(k-2) \pm \sqrt{(k-2)^2 - 4(2)(k+1)}}{2}
\]

\( k = \frac{12 \pm \sqrt{144 + 16}}{2} \)

\( k = \frac{12 \pm 16}{2} \Rightarrow k = 6 \) (6.32) \( 12.32 \) (5 pts)

\[
L_s = \frac{4(7^2)}{10.32} = \frac{67.75}{sec} (5 pts)
\]

\text{Non Oscillatory} (5 pts)

c.) Give your recommended design value of \( K \) based on minimum settling time with no oscillations.

\( k = 12.32 \) (5 pts)

d.) Compute the value of steady state error which will result with your selected value of \( K \).

\[
C_{ss} = \frac{12.325(2)}{12.325(2)} = 0.9249 = 97.50\% \text{ Error} (5 pts)
\]
e.) If the system has a point of marginal stability, tell what the frequency of oscillation will be there.

\[
\frac{C}{K} = \frac{k(s+2+\frac{1}{s})}{s^2-2s+2+k(s-2+\frac{1}{s})} - \frac{k(s^2+2s+1)}{s^3+(k-2)s^2+2(k+1)s+k}
\]

\[
S^3 - 2s - 2 + k
\]

\[
S^2 - k - 2
\]

\[
S^1 - 0
\]

\[
S^0 - k
\]

\[
\frac{1}{k-2} \left( k - (k-2)e_{k+1} \right) = -\frac{1}{k-2} \left( k - (2k^2-2k-4) \right) = \frac{2k^2-3k-4}{k-2}
\]

\[
k = \frac{3 \pm \sqrt{9+32}}{4} = 2.351 \quad (5 \text{ pt.})
\]

\[
0.351s^2 + 2.351 = 0
\]

\[
\omega = 2.58 \text{ rad/s} \quad (5 \text{ pt.})
\]

f.) Please calculate the steady-state error if \( K \) is changed to a value of 1, with the other values remaining the same, and the desired output is equal to the input.

\[
C_{ss} = \frac{k(1)}{k} = 1
\]

\[
\text{SS Err: } 0 \%
\]

(5 pt.)

3. (10 points) What values of \( K \) and \( \tau \) will cause the system below to settle at a final steady-state value of \( 2/3 \) in a time of 2 seconds?

\[
\frac{C_{ss}}{K} = \frac{K}{2s+1} = \frac{K}{2s+K} = \frac{\frac{K}{2}}{(\frac{K}{2}+\frac{1}{s})} = 1
\]

\[
\frac{K}{2s+1} = \frac{2}{3} \Rightarrow \frac{k}{1+k} = \frac{2}{3}
\]

\[
k = 2 \quad (5 \text{ pt.})
\]

\[
2 = \frac{3}{2} \frac{\tau}{\tau} \Rightarrow \frac{\tau}{\tau} = \frac{3}{2} \quad 5 \text{ pt.}
\]